4-4 PROVING TRIANGLES CONGRUENT-SSS, SAS (DAY 2)

SSS Congruence Postulate: If 3 sides of one △ are ≅ to the corresponding 3 sides of another △, then the △s are ≅.

SAS Congruence Postulate: If 2 sides and the included angle of one △ are ≅ to the corresponding 2 sides and included angle of another △, then the △s are ≅.

Example 1:

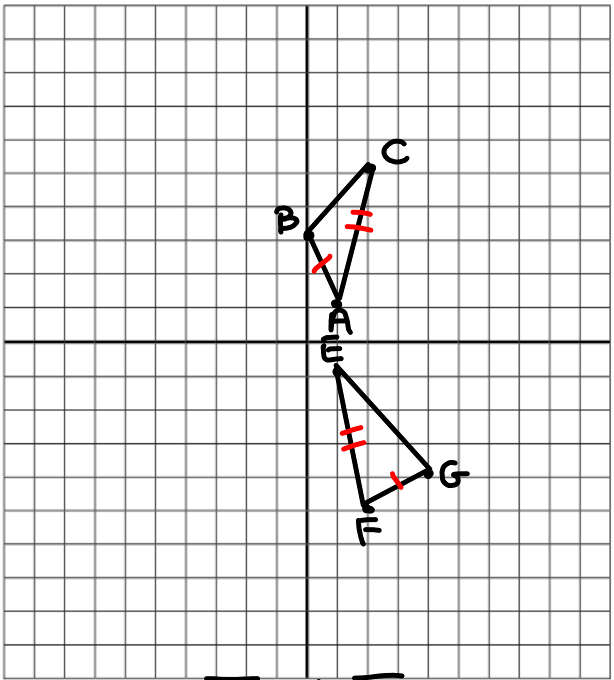
**EXTENDED RESPONSE** Triangle $ABC$ has vertices $A(1, 1)$, $B(0, 3)$, and $C(2, 5)$. Triangle $EFG$ has vertices $E(1, -1)$, $F(2, -5)$, and $G(4, -4)$.

**a.** Graph both triangles on the same coordinate plane.

**b.** Use your graph to make a conjecture as to whether the triangles are congruent. Explain your reasoning.

**c.** Write a logical argument using coordinate geometry to support the conjecture you made in part b.

(b) The △s do <u>not</u> appear to be ≅ to each other. (conjecture).




(c) use distance formula to find the lengths of each pair of corresponding sides.

$AB = \sqrt{(0-1)^2+(3-1)^2} = \sqrt{1+4} = \sqrt{5}$

$EF = \sqrt{(2-1)^2+(-5+1)^2} = \sqrt{1+16} = \sqrt{17}$

$BC = \sqrt{(2-0)^2+(5-3)^2} = \sqrt{4+4} = \sqrt{8}$

$FG = \sqrt{(4-2)^2+(-4+5)^2} = \sqrt{4+1} = \sqrt{5}$

$AC = \sqrt{(2-1)^2+(5-1)^2} = \sqrt{1+16} = \sqrt{17}$

$EG = \sqrt{(4-1)^2+(-4+1)^2} = \sqrt{9+9} = \sqrt{18}$

We have: $\overline{AB} \cong \overline{FG}$

$\overline{AC} \cong \overline{EF}$

BUT: $\overline{BC}$ not ≅ $\overline{EG}$

$\ncong$

So, △ABC is not ≅ to △EFG, because there is no SSS.

Example 2: Triangle JKL has vertices J(2, 5), K(1, 1), and L(5, 2). Triangle NPQ has vertices N(-3, 0), P(-7, 1), and Q(-4, 4).

a. Graph both triangles on the same coordinate plane.

b. Use your graph to make a conjecture as to whether the triangles are congruent. Explain your reasoning. The $\triangle$s appear to be congruent → have same shape + size.

c. Write a logical argument using coordinate geometry to support the conjecture you made in part b.

Try with transformations:

*idea* looks like

$r_{y\text{-axis}}$ → Translation

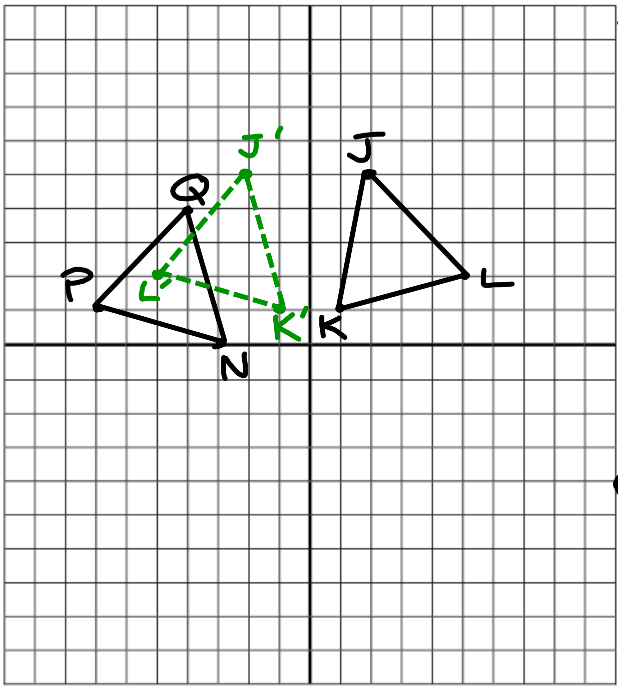
① Reflect $\triangle JKL$ over y-axis.

$r_{y\text{-axis}}\ (x,y) \to (-x,y)$

$J(2,5) \to J'(-2,5)$

$K(1,1) \to K'(-1,1)$

$L(5,2) \to L'(-5,2)$




② find the translation that maps $\triangle J'K'L'$ to $\triangle QNP$.

$(x,y) \to (x-2, y-1)$

$\langle -2,-1 \rangle$

$J'(-2,5) \to (-2-2, 5-1)$

$\to (-4,4) = Q$

$K'(-1,1) \to (-1-2, 1-1)$

$\to (-3,0) = N$

$L'(-5,2) \to (-5-2, 2-1)$

$\to (-7,1) = P$

$\triangle JKL \cong \triangle QNP$

by $T_{-2,-1} \circ r_{y\text{-axis}}$;

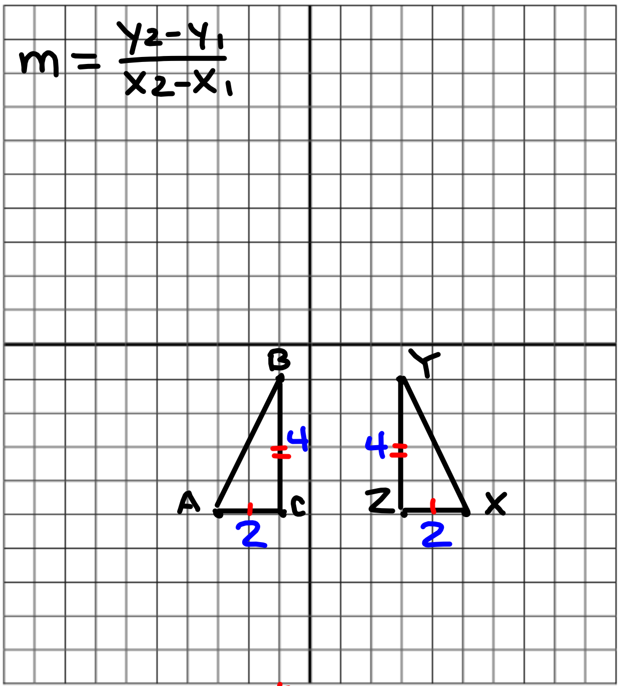
Translation and reflection are both congruence transformations.

Example 3: Triangle ABC has vertices A(-3, -5), B(-1, -1), and C(-1, -5). Triangle XYZ has vertices X(5, -5), Y(3, -1), and Z(3, -5).

a. Graph both triangles on the same coordinate plane.

b. Use your graph to make a conjecture as to whether the triangles are congruent. Explain your reasoning. The $\triangle$s appear to have same shape + size.

c. Write a logical argument using coordinate geometry to support the conjecture you made in part b.




$AC = 2,\ XZ = 2 \rightarrow \overline{AC} \cong \overline{XZ}$

$BC = 4,\ YZ = 4 \rightarrow \overline{BC} \cong \overline{YZ}$

prove $\angle C \cong \angle Z$.
Do slopes to show $\overline{AC} \perp \overline{BC}$ and $\overline{YZ} \perp \overline{XZ}$

slope $AC = \frac{-5 - -5}{-1 - -3} = \frac{0}{2} = 0$

slope $BC = \frac{-5 - -1}{-1 - -1} = \frac{-4}{0} =$ undefined

slope $= 0 \rightarrow$ horizontal
slope $=$ undefined $\rightarrow$ vertical
horizontal $\perp$ vertical $\rightarrow$
$\overline{AC} \perp \overline{BC}$

$\therefore m\angle C = 90$
↳ "therefore"

slope $XZ = \frac{-5 - -5}{3 - 5} = \frac{0}{-2} = 0$

slope $YZ = \frac{-5 - -1}{3 - 3} = \frac{-4}{0} =$ undefined

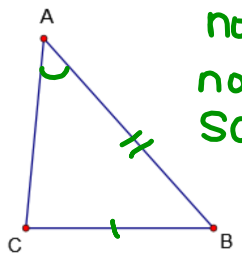
horizontal $\perp$ vertical $\rightarrow$
$\overline{XZ} \perp \overline{YZ}$
$\therefore m\angle Z = 90 \longrightarrow \angle C \cong \angle Z$

* $\triangle ABC \cong \triangle XYZ$ by SAS

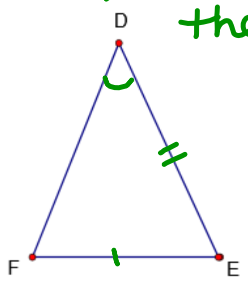
**WARNING!!**

Although SAS is a congruence postulate SSA is **NOT**.

→ donkey theorem



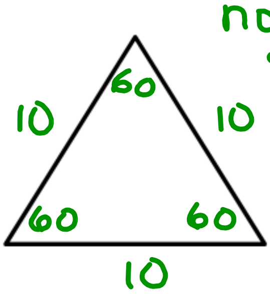

not ≅ → not the same shape.




GIVEN: $\overline{CB} \cong \overline{FE}$, $\overline{AB} \cong \overline{DE}$, and $\angle A \cong \angle D$.

Although SSS is a congruence postulate, AAA is **NOT**.

2 equilateral triangles.




not same size → not ≅